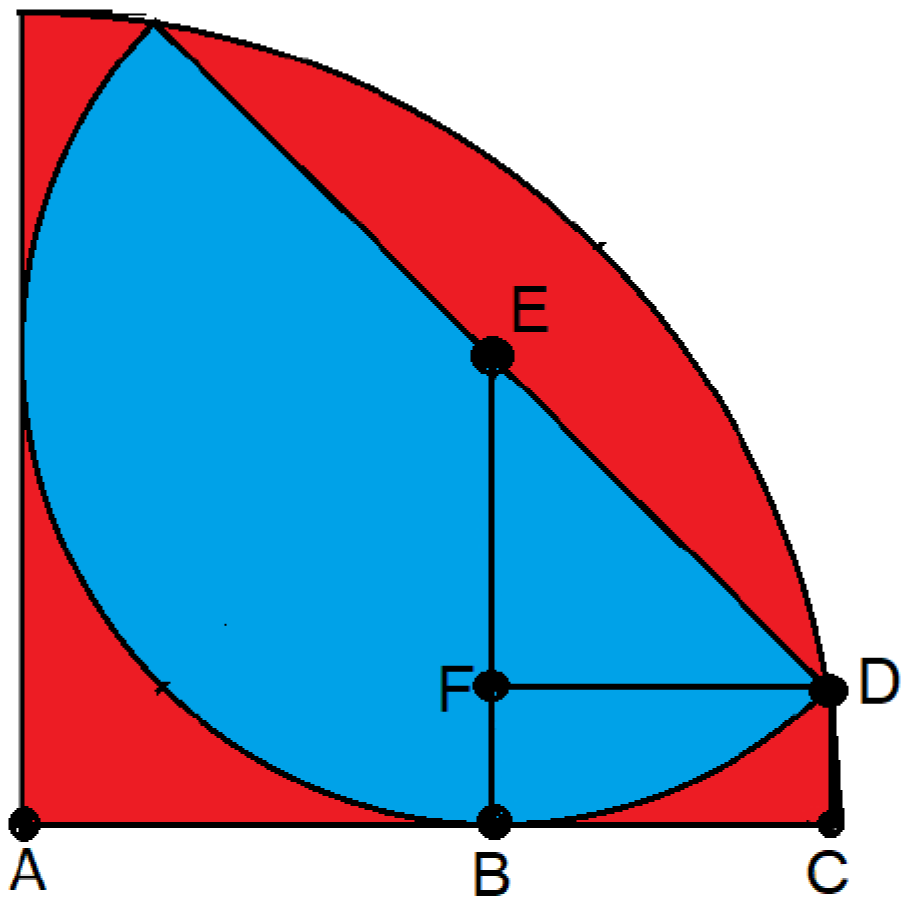
Problem 231 solution by ***Michael Shackleford of MathProblems.info***

Let's define some points, as shown in the following diagram:



- A -- Corner formed by the two flat sides of the quarter-circle.
- B -- Between A and C and directly below E, where the curve of the semi-circle touches the bottom edge of quarter-circle.
- C -- Directly under D on the bottom side of the quarter-circle. Note this is not on the curve of the quarter-circle.
- D -- Where the lower corner of the semi-circle and the curve of the quarter-circle touch.

- E -- Midpoint of the edge of the semi-circle.
- F -- Point between E and B, such that EFD is a right angle.

Let's let the distance between B and E, denoted as BE, equal to 1.

Since angle ABE is 90 degrees, AB also equals 1.
AB = 1, because BE is 1, and ABE is a right isosceles triangle.
AE = sqrt(2), by Pythagorean.
DE = 1, because AB=1 and they are both on the edge of the circle centered by E.
EF = DF = sqrt(2)/2, by Pythagorean, knowing the hypotenuse of that triangle is 1.
BF = DC = BC = EB - EF = 1 - sqrt(2)/2 = (2-sqrt(2))/2.
AC = 1 + sqrt(2)/2 = (2+sqrt(2))/2, because AC = AB+BC, which we both know.

We know AC and DC, so we can find AD by Pythagorean.

AD = sqrt[(AC)^2 + (DC)^2) = sqrt (((2+sqrt(2))/2)^2 + ((2-sqrt(2))/2)^2]
= sqrt[(4+4*sqrt(2)+2)/4 + (4-4*sqrt(2) + 2)/4] (using the FOIL method)
= sqrt[(4 + 2 + 4 + 2)/4]
= sqrt (12/4)
= sqrt(3) = radius of the quarter-circle

Given the radius of the semi-circle is 1, the area of the semi-circle is (1/2)*pi*1^2 = pi*(1/2)

Given the radius of the quarter-circle is sqrt(3), the area is quarter-circle is (1/4)*pi*(sqrt(3))^2 = pi * (3/4)

So, the ratio of the semi-circle to the quarter-circle is pi*(1/2)/(pi*(3/4)) = (2/4)/(3/4) = 2/3.