What if your opponent selects his number randomly?

Select z = 1/3 and your edge would be 1/6. You will win 4/9 of the time, lose 5/18 of the time, leaving 5/18 of the time for ties.

We can find a formula for the edge is by integrating the difference of the probability of a win and the probability of a loss over the unit interval and then finding the maximum of that expression.

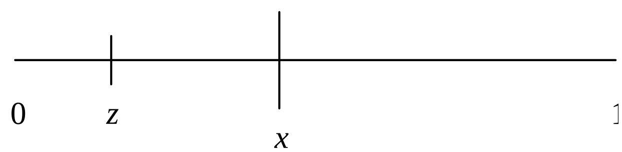
Let y = referees number (uniform on [0,1])
Let z = my choice of a number from some probability density function (pdf), $f(z)$
Let x = choice of opponent who selects number randomly (uniform on [0,1]).

Let W = Probability of win for player 1; we win when $x < z < y$ and when $z < y < x$.
Let L = Probability of loss for player 1; we lose when $x < y < z$ and when $z < x < y$.
Let T = Probability of tie for player 1; we tie when $y < x < z$ and when $y < z < x$.
And W + L + T = 1.

The edge equals W – L in this problem since ties are worth zero.

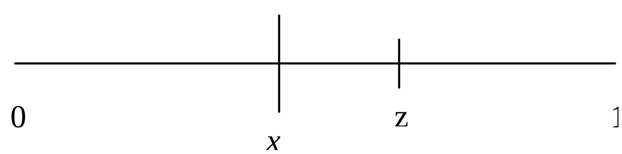
Generally we would have to solve double integrals to determine W, L, and T, integrating over all possible values of y and z, but since we know y is a uniform random variable, we can use the figures below to determine the necessary probabilities for y, depending on whether $z < x$ or $x < z$.

$z < x$:



y can be greater than x with probability $1 - x$, which would be a loss, and y can be between z and x with probability $x - z$, which would be a win.

$z > x$:



y can be greater than z with probability $1 - z$, which would be a win, and y can be between z and x with probability $z - x$, which would be a loss.

$$\int_0^z [(1-z)-(z-x)]dz + \int_z^1 [(x-z)-(1-x)]dz = z - \frac{3}{2}z^2$$

$$\frac{d}{dz}\left(z - \frac{3}{2}z^2\right) = 1 - 3z = 0 \Rightarrow z = \frac{1}{3}$$ when the function achieves its maximum value of 1/6.