Coin Toss Problem

Richard Tucker

Problem:

Given a coin with probability p of landing on heads after a flip, what is the probability that the number of heads will ever equal the number of tails assuming an infinite number of flips?

Solution:

Think of this problem instead as one of a random walk along a number line. Let 0 be the starting point, p be the probability of moving to the right, and q be the probability of moving to the left.

Let B=probability of ever moving one to the left from where you are.

Let A=probability of revisiting current square from the right.

$B = q + Aq + A^2q + A^3q + ... = q/(1-A)$.

A=pB --> B=A/p --> B=A/(1-q).

q/(1-A) = A/(1-q) --> A=q,1-q.

However, A must be less than or equal to both p and q, thus the reasonable solution is A=min(q,1-q).

Redo the above only reverse the words left and right and A will still equal min(q,1-q). One ramification of this is that the probability of revisiting 0 is the same from both the left as the right. This stands to reason since any path has a mirror image on the other side of equal probability. So the answer is 2*min(q,1-q). Where I am a little uncomfortable is dismissing the other solution of A. I believe I can do so but can not put into words why.